

SNUTP11-006
HRI/IST/1105
KUNS-2349

BPS solutions in ABJM theory and Maximal Super Yang-Mills on $\mathbf{R} \times \mathbf{S}^2$

Bobby Ezhuthachan^{1)*}, Shinji Shimasaki^{1),2)†} and Shuichi Yokoyama^{3),4)‡}

¹⁾ *Harish-Chandra Research Institute, Chhatnag Rd, Jhansi, Allahabad 211019, India*

²⁾ *Department of Physics, Kyoto University, Kyoto 606-8592, Japan*

³⁾ *Department of Physics and Astronomy and Center for Theoretical Physics,
Seoul National University, Seoul 51-747, Korea*

⁴⁾ *Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*

Abstract

We investigate BPS solutions in ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. We find new BPS solutions, which have nonzero angular momentum as well as nontrivial configurations of fluxes. Applying the “Higgsing procedure” of arXiv:0803.3218 around a 1/2-BPS solution of ABJM theory, one obtains $\mathcal{N} = 8$ super Yang-Mills (SYM) on $\mathbf{R} \times \mathbf{S}^2$. We also show that other BPS solutions of the SYM can be obtained from BPS solutions of ABJM theory by this higgsing procedure.

* e-mail address : bobby(at)hri.res.in

† e-mail address : shinji(at)gauge.scphys.kyoto-u.ac.jp

‡ e-mail address : yokoyama(at)phys.snu.ac.kr

1 Introduction

Superconformal Chern-Simons-matter (CSM) theories have been studied with considerable interest over the past few years. These theories have been studied in the context of M-theory and their possible relevance to the world-volume theory of multiple M2-branes was first discussed in [1]. The first explicit Lagrangian of such a CSM theory was BLG theory [2–5]. This was a maximally supersymmetric $\mathcal{N} = 8$ superconformal theory of fixed rank $SU(2) \times SU(2)$ coupled to matter fields transforming in the bi-fundamental of the two $SU(2)$'s. The Chern-Simons terms of the two $SU(2)$'s come with a relative negative sign. Even though the relevance of the BLG theory to M2-brane theory is not understood, CSM theories with lesser supersymmetry, sharing some of the above mentioned features of the BLG theory, have been proposed as the world-volume description of M2-branes in various backgrounds. In particular, a certain $\mathcal{N} = 6$ superconformal CSM theory - ABJM theory - was proposed as the world-volume theory of multiple M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$, where k is the Chern-Simons level [6]. For $k = 1, 2$, ABJM theory has $\mathcal{N} = 8$ supersymmetries even though in the classical Lagrangian only $\mathcal{N} = 6$ supersymmetries are manifest. The enhanced symmetry generators are realized in terms of monopole operators [6–8].

Several checks have been done for this proposal. Firstly the moduli space of the theory has been shown to have the right geometry. In the case of ABJM theory, for instance, the moduli space is $\mathbb{C}^4/\mathbb{Z}_k$. Tests beyond getting the right moduli space have also been done. This includes the computation of the superconformal index of the theory and matching with results from supergravity [9–13]. Several CSM theories have been proposed to describe M2-branes in other backgrounds [14–22].

One of the first checks of the relevance of these CSM theories to M-theory was performed in [23, 24]. In the case of M2-branes on $\mathbb{C}^4/\mathbb{Z}_k$, one can consider a limit in which we take the branes far away from the orbifold fixed point and simultaneously take small orbifold angle. In this limit the orbifold geometry can be well approximated by $\mathbf{S}^1 \times \mathbf{R}^7$. This is the limit in which the M2-branes should be approximated by D2-branes, and therefore the CSM theory should be approximated by a super Yang-Mills theory (SYM). Mukhi and Papageorgakis gave a field theory realization of this picture in BLG theory¹.

¹ Even though the geometry of the moduli space of BLG theory is more complicated than $\mathbb{C}^4/\mathbb{Z}_k$, the Higgsing procedure still leads to SYM.

By giving a vev to a scalar field, and taking the large v and large k limit with $\frac{2\pi v^2}{k} = g_{ym}^2$ held constant as the gauge coupling, it was shown that the CSM theory is approximated by $\mathcal{N} = 8$ SYM on flat spacetime. This procedure was called the “novel Higgs mechanism”. This was first done in the context of the maximally supersymmetric $\mathcal{N} = 8$ BLG theory but carries over for ABJM theory as well [6].

For the abelian versions of the theories, corresponding to a single D2 brane and single M2 brane, it can be explicitly seen that the ABJM at $k = 1$ can be rewritten as the SYM by simply compactifying one of the eight-scalar fields and dualizing it into a gauge field. Of course, for the non-abelian theory, it is not possible to carry out a compactification directly at the level of the classical Lagrangian because the translation invariance along the transverse directions is not manifest in the Lagrangian. Also, since the SYM is interacting, one expects the $SO(8)$ invariance to be manifest only at the strongly coupled IR fixed point of the SYM². Therefore the Higgsing procedure is the only way in which one can see the M2 to D2 connection at the level of the classical Lagrangian.

Since ABJM theory is conformal there exists a conformal map which maps ABJM theory on flat spacetime to that on $\mathbf{R} \times \mathbf{S}^2$. Under this map the vacua of ABJM theory get mapped to time-dependent 1/2-BPS solutions on $\mathbf{R} \times \mathbf{S}^2$ [27]. The novel Higgs mechanism was carried out around the vacua of the CSM theory on flat space and resulted in $\mathcal{N} = 8$ SYM. It is worth asking what happens when we carry out the analogous procedure of the novel Higgs mechanism about the corresponding solutions of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. In this case, it is naturally expected that we obtain $\mathcal{N} = 8$ SYM³ on $\mathbf{R} \times \mathbf{S}^2$, which preserves $SU(2|4)$ symmetry (16 supersymmetries) and has been studied previously in the context of the plane wave (BMN) matrix model [28], gauge/gravity duality [29, 30] and the large- N reduction of $\mathcal{N} = 4$ SYM on $\mathbf{R} \times \mathbf{S}^3$ [30]. Thermodynamic aspects of this SYM was studied in [31] while aspects related to integrability was studied in [32].

In this paper, we first solve for BPS configurations in ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. In particular, we find general BPS solutions for diagonal configurations. Interestingly, the BPS solutions have non-trivial (t, θ, φ) -dependence on $\mathbf{R} \times \mathbf{S}^2$ with nonzero angular

² However, in [25], it was shown that even in the non-abelian case the enhanced $SO(8)$ invariance can be seen manifestly at the level of scattering amplitudes of the SYM. See also [26].

³ $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ is no longer related to the $\mathcal{N} = 8$ SYM on flat space because the theory is not conformal.

momentum on \mathbf{S}^2 as well as non-trivial flux, not only “magnetic flux” but also “electric flux”, turned on. We then show that carrying out the Higgsing procedure around a 1/2-BPS solution of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ leads to $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$. In this process, as in the flat space case, we observe an enhancement of the supersymmetry and the R -symmetry, from 12 and $SU(3)$ to 16 and $SU(4)$, respectively⁴. We also comment on the mechanism of this enhancement. Furthermore we show that the theory around a nontrivial vacuum and a 1/2-BPS solution of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ is also obtained by Higgsing the theory around another 1/2-BPS solution and a 1/4-BPS solution, respectively, of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$.

The organization of this paper is as follows. In section 2, we write down the action, equations of motion and supersymmetries of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. In section 3, we solve for specific 1/2-BPS and 1/4-BPS solutions of this theory. In section 4, we then show that higgsing around a 1/2-BPS solution of ABJM on $\mathbf{R} \times \mathbf{S}^2$ leads to the $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ and make some comment on the symmetry enhancement. We also show that theories expanded around a nontrivial vacuum and a 1/2-BPS solution of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ are obtained from ABJM theory. Section 5 is devoted to summary and discussion. There are four appendices in which we collect our notations and conventions used in the paper, give some details about the BPS solutions of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$, present the action, supersymmetry transformations and vacuum solutions of the $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ and give some details about the representation of the R -symmetry of fermions in ABJM theory and SYM.

2 ABJM on $\mathbf{R} \times \mathbf{S}^2$

In this section we write down the action, equations of motion and supersymmetry transformations of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ with Minkowski signature $(-++)$.

The field content of ABJM theory is the following: two gauge fields $A^{(1)}$ and $A^{(2)}$ associated with the gauge group $U(N) \times U(N)$, bi-fundamental scalars Y^A and their superpartners ψ_A ($A = 1, 2, 3, 4$), which are $(1+2)$ -dimensional Majorana spinors. The global symmetry of this theory is the superconformal symmetry $OSp(6|4)$ and a $U(1)$

⁴ This is the supersymmetry and global symmetry preserved by the 1/2-BPS solution about which we “Higgs”.

(baryon) symmetry, denoted by $U(1)_b$. $OSp(6|4)$ includes the $(1+2)$ -dimensional conformal group $SO(2,3)$ and R -symmetry $SU(4)$ as bosonic subgroups. Y^A (ψ_A) transforms as the (anti-)fundamental representation of $SU(4)$ and carries charge $-1(+1)$ under $U(1)_b$.

The action of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ is given by

$$\begin{aligned}
S = \int dt \frac{d\Omega_2}{\mu^2} \text{Tr} & \left[\frac{k}{4\pi} \epsilon^{mnp} \left(A_m^{(1)} \partial_n A_p^{(1)} + \frac{2i}{3} A_m^{(1)} A_n^{(1)} A_p^{(1)} - A_m^{(2)} \partial_n A_p^{(2)} - \frac{2i}{3} A_m^{(2)} A_n^{(2)} A_p^{(2)} \right) \right. \\
& - D_m Y_A^\dagger D^m Y^A - \frac{\mu^2}{4} Y_A^\dagger Y^A + i \psi^\dagger A \gamma^a D_a \psi_A \\
& + \frac{4\pi^2}{3k^2} \left(Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right) \\
& + \frac{2\pi i}{k} \left(\psi_A \psi^\dagger A Y^B Y_B^\dagger - \psi^\dagger A \psi_A Y_B^\dagger Y^B + 2\psi^\dagger A \psi_B Y_A^\dagger Y^B - 2\psi_A \psi^\dagger B Y^A Y_B^\dagger \right) \\
& \left. + \frac{2\pi i}{k} \left(\epsilon_{ABCD} \psi^\dagger A Y^B \psi^\dagger C Y^D - \epsilon^{ABCD} \psi_A Y_B^\dagger \psi_C Y_D^\dagger \right) \right]. \tag{2.1}
\end{aligned}$$

where $m, n, p \dots$ run over the world-volume coordinates t, θ, φ and $a, b, \dots = 1, 2, 3$ are corresponding local Lorentz indices. The upper and lower A, B, \dots are indices of $\mathbf{4}$ and $\bar{\mathbf{4}}$, respectively, of $SU(4)$ and run $1, 2, 3, 4$. $k(= 1, 2, \dots)$ is the Chern-Simons coupling and μ^{-1} is the radius of S^2 . γ^a ($a = 1, 2, 3$) are gamma matrices of $SO(1, 2)$, which satisfy $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ with $\eta^{ab} = \text{diag}(-1, +1, +1)$. The mass term of the scalar field comes from the coupling to the background curvature. Covariant derivatives take the following form

$$\begin{aligned}
D_m Y^A &= \partial_m Y^A + i A_m^{(1)} Y^A - i Y^A A_m^{(2)}, \\
D_m \psi_A &= \nabla_m \psi_A + i A_m^{(1)} \psi_A - i \psi_A A_m^{(2)} \\
&= \partial_m \psi_A + \frac{1}{4} \omega_{mab} \gamma^{ab} \psi_A + i A_m^{(1)} \psi_A - i \psi_A A_m^{(2)}. \tag{2.2}
\end{aligned}$$

where ω_{ab} is the spin connection of $\mathbf{R} \times \mathbf{S}^2$. In appendix A, we gather our conventions of the metric and the spinor used in this paper. Equations of motion for the bosonic fields with $\psi_A = 0$, which are relevant for the following discussion, are given by

$$\begin{aligned}
\epsilon^{abc} \frac{k}{4\pi} F_{bc}^{(1)} &= i \left(Y^A D^a Y_A^\dagger - D^a Y^A Y_A^\dagger \right), \\
\epsilon^{abc} \frac{k}{4\pi} F_{bc}^{(2)} &= i \left(D^a Y_A^\dagger Y^A - Y_A^\dagger D^a Y^A \right), \\
\left(D_a D^a - \frac{\mu^2}{4} \right) Y^A &= -\frac{4\pi^2}{k^2} \left(Y^B Y_B^\dagger Y^C Y_C^\dagger Y^A + Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4Y^B Y_C^\dagger Y^A Y_B^\dagger Y^C \right)
\end{aligned}$$

$$-2Y^BY_B^\dagger Y^AY_C^\dagger Y^C - 2Y^AY_B^\dagger Y^CY_C^\dagger Y^B - 2Y^BY_C^\dagger Y^CY_B^\dagger Y^A). \quad (2.3)$$

We can show that the action (2.1) is invariant under the following supersymmetry transformations⁵

$$\begin{aligned} \delta Y^A &= -i\xi^{AB}\psi_B, \\ \delta Y_A^\dagger &= -i\psi^{\dagger B}\xi_{AB}, \\ \delta\psi_A &= -\gamma^m\xi_{AB}D_mY^B - \frac{2\pi}{k}Q^B{}_A{}^C\xi_{BC} - \frac{1}{3}Y^B\gamma^m\nabla_m\xi_{AB}, \\ \delta\psi^{\dagger A} &= \xi^{AB}\gamma^mD_mY_B^\dagger - \frac{2\pi}{k}(Q^B{}_A{}^C)^\dagger\xi^{BC} + \frac{1}{3}Y_B^\dagger\nabla_m\xi^{AB}\gamma^m, \\ \delta A_m^{(1)} &= -\frac{2\pi}{k}\left[Y^B\psi^{\dagger A}\gamma_m\xi_{AB} + \xi^{AB}\gamma_m\psi_AY_B^\dagger\right], \\ \delta A_m^{(2)} &= -\frac{2\pi}{k}\left[\psi^{\dagger A}\gamma_m\xi_{AB}Y^B + Y_B^\dagger\xi^{AB}\gamma_m\psi_A\right], \end{aligned} \quad (2.4)$$

where

$$Q^B{}_A{}^C \equiv T^B{}_A{}^C - \frac{1}{2}\delta_A^CT^B{}_D{}^D + \frac{1}{2}\delta_A^BT^C{}_D{}^D, \quad T^B{}_A{}^C \equiv Y^BY_A^\dagger Y^C - Y^CY_A^\dagger Y^B. \quad (2.5)$$

ξ_{AB} are supersymmetry parameters, which are $(1+2)$ -dimensional Majorana spinors and antisymmetric in A and B (i.e. **6** of $SU(4)_R$), $\xi_{AB} = -\xi_{BA}$, and satisfy the conformal Killing spinor equations,

$$\nabla_a\xi_{AB} = \pm i\frac{\mu}{2}\gamma_a\gamma^0\xi_{AB}. \quad (2.6)$$

Hereafter we denote ξ_{AB} satisfying the upper and lower signs in (2.6) by $\xi_{AB}^{(+)}$ and $\xi_{AB}^{(-)}$, respectively. $\xi^{(\pm)AB}$ is the complex conjugate of $\xi_{AB}^{(\pm)}$ and satisfy

$$\xi^{(\pm)AB} \equiv (\xi_{AB}^{(\pm)})^* = -\frac{1}{2}\epsilon^{ABCD}\xi_{CD}^{(\mp)}. \quad (2.7)$$

So, $\xi_{AB}^{(\pm)}$ are related to the complex conjugate of $\xi_{AB}^{(\mp)}$. One can easily solve (2.6) as

$$\xi_{AB}^{(\pm)} = e^{\pm i\frac{\mu t}{2}}e^{\mp i\gamma^2\frac{\theta}{2}}e^{\gamma^0\frac{\phi}{2}}\eta_{AB}^{(\pm)} \quad (2.8)$$

where $\eta_{AB}^{(\pm)}$ are constant spinors. Thus the action (2.1) possesses 24 supersymmetries.

⁵ For $k = 1, 2$, there are additional supersymmetries which are not manifest in the Lagrangian.

3 BPS solutions of ABJM on $\mathbf{R} \times \mathbf{S}^2$

In this section, we find specific BPS solutions of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. BPS solutions, in general, are obtained by solving $\delta\psi_A = 0$ as well as the equations of motion with $\psi_A = 0$. Since it is difficult to solve the equations generically, we look for solutions with diagonal configuration in the $U(N) \times U(N)$ theory. For these solutions, $Q^B{}_A{}^C = 0$. Therefore each diagonal component is basically a BPS solution of the $U(1) \times U(1)$ theory. The BPS equations can be easily solved with this assumption. In the following, we give particular BPS solutions, which are 1/2-BPS and 1/4-BPS solutions for $U(1) \times U(1)$ ABJM theory when $k > 2$. They are determined by $\delta\psi_A = 0$, where $\delta\psi_A$ is given in (2.4). Other BPS solutions are summarized in appendix B.

3.1 1/2-BPS solution

We first look for 1/2-BPS solutions of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ [27, 33, 34]. Let us consider the equation given by $\delta\psi_A = 0$ in $U(1) \times U(1)$ ABJM theory,

$$-\gamma^m \xi_{AB}^{(\pm)} D_m Y^B \mp i \frac{\mu}{2} Y^B \gamma^0 \xi_{AB}^{(\pm)} = 0, \quad (3.1)$$

where $\xi_{AB}^{(\pm)}$ is explicitly given in (2.8). Since the equations of motion for the gauge fields imply $F_{mn}^{(1)} = F_{mn}^{(2)}$, we can take a gauge in which

$$A_m^{(1)} = A_m^{(2)}, \quad (3.2)$$

so that D_m becomes ∂_m in (3.1). Now, we look for BPS solutions preserving $SU(3)$ of the $SU(4)$ R -symmetry. Such a configuration is obtained by imposing

$$\begin{aligned} \eta_{A'B'}^{(+)} &= 0, & \eta_{A'4}^{(+)} &\neq 0, \\ \eta_{A'4}^{(-)} &= 0, & \eta_{A'B'}^{(-)} &\neq 0 \end{aligned} \quad (3.3)$$

where $A', B', \dots = 1, 2, 3$ and the second line of (3.3) is the complex conjugate of the first line. This is a 1/2-BPS condition. Then, (3.1) reduces to the equations for the scalars

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ (\partial_t + i \frac{\mu}{2}) Y^4 &= 0, & \partial_\theta Y^4 &= \partial_\varphi Y^4 = 0. \end{aligned} \quad (3.4)$$

Therefore, a 1/2-BPS solution for the scalar fields is given by

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ Y^4 &= v e^{-i\frac{\mu}{2}t}, \end{aligned} \quad (3.5)$$

where v is a complex constant. This solution breaks $SU(4)$ R -symmetry to $SU(3)$. It turns out from the equations of motion of the gauge fields in (2.3) that the gauge fluxes take the form

$$\begin{aligned} F_{01}^{(1)} &= F_{01}^{(2)} = F_{02}^{(1)} = F_{02}^{(2)} = 0, \\ F_{12}^{(1)} &= F_{12}^{(2)} = \frac{2\pi\mu}{k}|v|^2. \end{aligned} \quad (3.6)$$

Flux quantization condition;

$$\frac{1}{2\pi} \int \frac{d\Omega}{\mu^2} F_{12}^{(i)} \in \mathbf{Z}. \quad (3.7)$$

leads to the quantization of v ;

$$\frac{4\pi}{\mu k} |v|^2 = 2q \in \mathbf{Z}_{\geq 0}, \quad (3.8)$$

where $q \in \mathbf{Z}_{\geq 0}/2$. One can easily solve (3.6) locally in terms of gauge fields by introducing two patches on S^2 ;

$$\begin{aligned} A_0^{(1)} &= A_0^{(2)} = 0, \\ A_1^{(1)} &= A_1^{(2)} = 0, \\ A_2^{(1)} &= A_2^{(2)} = \frac{2\pi|v|^2}{k} \frac{\pm 1 - \cos \theta}{\sin \theta} = \mu q \frac{\pm 1 - \cos \theta}{\sin \theta}, \end{aligned} \quad (3.9)$$

where we have taken $A_0^{(1)} = A_0^{(2)} = A_1^{(1)} = A_1^{(2)} = 0$ gauge. The upper and lower signs in the third line correspond to the region I ($0 \leq \theta < \pi$) and the region II ($0 < \theta \leq \pi$), respectively. For each patch, gauge fields are well-defined. This gauge field configuration is nothing but the Dirac monopole with the monopole charge q . In the overlap region, the configurations on the region I and the region II are related by the gauge transformation

$$U_{\text{II} \rightarrow \text{I}} = \exp \left\{ i \frac{4\pi}{\mu k} |v|^2 \cdot \varphi \right\} = \exp \{ i 2q \varphi \}, \quad (3.10)$$

which is single value since $q \in \mathbf{Z}/2$.

As discussed in [6], even after gauge fixing ABJM theory, there is a discrete redundant gauge symmetry left, which results in the following identification of scalar fields:

$$Y^A \sim e^{2\pi i/k} Y^A. \quad (3.11)$$

For the 1/2-BPS solutions (3.5) and (3.9), we can calculate the energy E and the R -charge J_4 (the charge corresponding to the rotation of the phase of Y^4);

$$\begin{aligned} E &= \int \frac{d\Omega}{\mu^2} \left(|\partial_t Y^A|^2 + |\nabla_{a'} Y^A|^2 + \frac{\mu^2}{4} |Y^A|^2 \right) = \mu k q, \\ J_4 &= \int \frac{d\Omega}{\mu^2} \left(-i Y^4 \partial_t Y_4^\dagger + i \partial_t Y^4 Y_4^\dagger \right) = 2kq, \end{aligned} \quad (3.12)$$

where $a' = 1, 2$. Note that the solution saturates the following BPS bound⁶

$$E = \frac{\mu}{2} J_4. \quad (3.13)$$

3.2 1/4-BPS solution

Next, we will find 1/4-BPS solutions. In addition to the 1/2-BPS condition (3.3) we further impose the following conditions

$$\begin{aligned} i\gamma^0 \eta_{A'4}^{(+)} &= \eta_{A'4}^{(+)}, \\ i\gamma^0 \eta_{A'B'}^{(-)} &= -\eta_{A'B'}^{(-)}, \end{aligned} \quad (3.14)$$

where the second condition is the complex conjugate of the first, so this gives rise to a 1/4-BPS condition. In this case, (2.8) becomes

$$\begin{aligned} \xi_{A'4}^{(+)} &= e^{i\frac{\mu t}{2}} e^{-i\frac{\phi}{2}} \left(\cos \frac{\theta}{2} + \gamma^1 \sin \frac{\theta}{2} \right) \eta_{A'4}^{(+)}, \\ \xi_{A'B'}^{(-)} &= e^{-i\frac{\mu t}{2}} e^{i\frac{\phi}{2}} \left(\cos \frac{\theta}{2} + \gamma^1 \sin \frac{\theta}{2} \right) \eta_{A'B'}^{(-)}. \end{aligned} \quad (3.15)$$

Substituting this into (3.1), we obtain the following conditions for the scalars

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ \partial_t Y^4 + i\frac{\mu}{2} Y^4 - \mu \partial_\varphi Y^4 &= 0, \\ \partial_\theta Y^4 + i \cot \theta \partial_\varphi Y^4 &= 0. \end{aligned} \quad (3.16)$$

⁶ The $\frac{1}{2}$ in the right-hand side is due to our R -charge assignment.

It is easily seen that $Y^4 \sim \sin^p \theta e^{ip\varphi} e^{-i(p+\frac{1}{2})\mu t}$ solves the above equation as well as the equation of motion. So the general solution of the scalar fields is given by

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ Y^4 &= \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} v_p \sin^p \theta e^{ip\varphi} e^{-i(p+\frac{1}{2})\mu t}, \end{aligned} \quad (3.17)$$

where n is an integer in the range of $0 \leq n \leq k-1$ and v_p are complex constants. When p is an integer, $\sin^p \theta e^{ip\varphi}$ is the spherical Harmonics of $l = m = p$, $Y_{pp}(\theta, \varphi)$. Here we have chosen p in such a way that the solution is regular at $\theta = 0, \pi$ and single-valued with (3.11) under the shift $\varphi \rightarrow \varphi + 2\pi$. As in the 1/2-BPS case, the 1/4-BPS solution (3.17) breaks $SU(4)$ R -symmetry to $SU(3)$. From the equations of motion of the gauge fields in (2.3), one can compute the gauge fluxes as

$$\begin{aligned} F_{12}^{(1)} &= F_{12}^{(2)} = \frac{2\pi\mu}{k} \sum_{p,p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (p+p'+1) v_p (v_{p'})^* \sin^{p+p'} \theta e^{i(p-p')(\varphi-\mu t)}, \\ F_{01}^{(1)} &= F_{01}^{(2)} = \frac{2\pi\mu}{k} \sum_{p,p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (p+p') v_p (v_{p'})^* \sin^{p+p'-1} \theta e^{i(p-p')(\varphi-\mu t)}, \\ F_{02}^{(1)} &= F_{02}^{(2)} = \frac{2\pi\mu i}{k} \sum_{p,p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (p-p') v_p (v_{p'})^* \cos \theta \sin^{p+p'-1} \theta e^{i(p-p')(\varphi-\mu t)}. \end{aligned} \quad (3.18)$$

Thus, in the general 1/4-BPS solutions determined by (3.3) and (3.14), in contrast to the 1/2-BPS case, not only $F_{12}^{(i)}$ but also $F_{0a'}^{(i)}$ ($a' = 1, 2$) are nonzero and furthermore they have nontrivial (t, θ, φ) dependence. The quantization condition of the flux requires

$$\frac{2\pi}{\mu k} \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} 2^{2p+1} \frac{\Gamma(p+1)^2}{\Gamma(2p+1)} |v_p|^2 = 2q \in \mathbf{Z}_{\geq 0}, \quad (3.19)$$

where $q \in \mathbf{Z}_{\geq 0}/2$. So v_p are given by

$$v_p = \frac{e^{i\alpha_p}}{c_p} \sqrt{\frac{\mu k q_p}{2\pi}}, \quad (3.20)$$

where

$$c_p = \sqrt{\frac{2^{2p} \Gamma(p+1)^2}{\Gamma(2p+1)}}, \quad (3.21)$$

α_p are real constants and q_p are real constants with $\sum_p q_p = q$. As in the 1/2-BPS case, (3.18) can be solved in terms of the gauge field with a gauge in which $A_1^{(1)} = A_1^{(2)} = 0$ as

$$\begin{aligned}
A_0^{(1)} &= A_0^{(2)} \\
&= \frac{2\pi}{k} \sum_{p \neq p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (p + p') v_p (v_{p'})^* e^{i(p-p')(\varphi - \mu t)} \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-\frac{p+p'}{2} + r}{r} (\mp 1 + \cos^{2r+1} \theta) \\
&\quad + \frac{2\pi}{k} \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} 2p |v_p|^2 \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-p+r}{r} \cos^{2r+1} \theta, \\
A_1^{(1)} &= A_1^{(2)} = 0, \\
A_2^{(1)} &= A_2^{(2)} = \frac{2\pi}{k} \sum_{p, p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (p + p' + 1) v_p (v_{p'})^* e^{i(p-p')(\varphi - \mu t)} \\
&\quad \times \frac{1}{\sin \theta} \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-\frac{p+p'}{2} + r - 1}{r} (\pm 1 - \cos^{2r+1} \theta),
\end{aligned} \tag{3.22}$$

where $\binom{a}{b}$ is the binomial coefficient. The upper and lower signs correspond to the region I ($0 \leq \theta < \pi$) and the region II ($0 < \theta \leq \pi$) on S^2 , respectively. Since all components of the field strength are nonzero and take the nontrivial form, in the present gauge, not only $A_2^{(i)}$ but also $A_0^{(i)}$ are nonzero and involve the t and φ -dependence as well as the θ -dependence. (The θ -dependence in $A_2^{(i)}$ seems to be a (higher order) generalization of the monopole configuration.) The patch-dependence of $A_0^{(i)}$ is introduced so that $A_0^{(i)}$ does not have φ -dependence at $\theta = 0$ and π . Thus, on each patch, gauge fields are well-defined. In the overlap region, one can transform the configurations of the gauge fields (3.22) from one to the other by the transition function

$$U_{\text{II} \rightarrow \text{I}} = \exp \left\{ \frac{4\pi i}{\mu k} \sum_{p \neq p' \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} 2^{p+p'} \frac{\Gamma(\frac{p+p'}{2} + 1)^2}{\Gamma(p+p'+1)} v_p (v_{p'})^* \frac{e^{i(p-p')(\varphi - \mu t)}}{i(p-p')} + 2iq\varphi \right\}. \tag{3.23}$$

Note that

$$\begin{aligned}
\sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-p+r-1}{r} &= \frac{2^{2p} \Gamma(p+1)^2}{\Gamma(2p+2)} \\
&= \frac{2p}{2p+1} \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-p+r}{r}
\end{aligned} \tag{3.24}$$

The solution with $n = 0$ and $v_l = 0$ for $l \geq 1$ is the 1/2-BPS solution discussed in the previous subsection.

Finally, we calculate charges for the 1/4-BPS solutions. In addition to the energy and the R -charge computed in the 1/2-BPS case, 1/4-BPS solutions have nonzero momentum along φ direction,

$$\begin{aligned} E &= \int \frac{d\Omega}{\mu^2} \left(|\partial_t Y^A|^2 + |\nabla_{a'} Y^A|^2 + \frac{\mu^2}{4} |Y^A|^2 \right) = 2\pi \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} (2p+1) c_p^2 |v_p|^2, \\ J_4 &= \int \frac{d\Omega}{\mu^2} \left(-i Y^4 \partial_t Y_4^\dagger + i \partial_t Y^4 Y_4^\dagger \right) = 2kq, \\ P_\varphi &= \int \frac{d\Omega}{\mu^2} \left(-\partial_t Y^A \partial_\varphi Y_A^\dagger + \partial_\varphi Y^A \partial_t Y_A^\dagger \right) = \frac{2\pi}{\mu} \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} 2p c_p^2 |v_p|^2. \end{aligned} \quad (3.25)$$

So the 1/4-BPS solution satisfies the following BPS bound

$$E = \mu \left(\frac{1}{2} J_4 + P_\varphi \right). \quad (3.26)$$

4 SYM on $\mathbf{R} \times \mathbf{S}^2$ from ABJM on $\mathbf{R} \times \mathbf{S}^2$

In this section we “Higgs” ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ around a 1/2-BPS solution following the procedure first discussed in [23]. In [23], Mukhi and Papageorgakis had shown that one can obtain $\mathcal{N} = 8$ SYM from BLG theory on \mathbf{R}^3 by expanding it around a vacuum $Y^A = \delta^{A4} v \mathbf{1}_N$ and taking the limit in which $v \rightarrow \infty$ and $k \rightarrow \infty$ with v^2/k fixed. This procedure was called the “novel Higgs mechanism”.

Here we will show that when a similar procedure is carried out around a 1/2-BPS solution in ABJM theory on $\mathbf{R} \times \mathbf{S}^2$, the action reduces to $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$, which has interesting features such as the existence of many discrete vacua, a mass gap and $SU(2|4)$ symmetry (16 supercharges)⁷. Some details of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ are summarized in appendix C. Since $\mathcal{N} = 8$ SYM in three dimensions is not conformal, the theory on $\mathbf{R} \times \mathbf{S}^2$ is not related to that on \mathbf{R}^3 in any simple way, unlike ABJM theory. It should be noted that the theory expanded around a 1/2-BPS solution of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ has 12 supersymmetries and $SU(3)$ R -symmetry while $\mathcal{N} = 8$ SYM on

⁷In the abelian case, the relation between the theory of a single M2-brane and the abelian SYM on $\mathbf{R} \times \mathbf{S}^2$ has been discussed in [35].

$\mathbf{R} \times \mathbf{S}^2$ has 16 supersymmetries and $SU(4)$ R -symmetry, so in the Higgsing we will see the enhancement of the R -symmetry as well as the number of supersymmetries.

4.1 $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ around trivial vacuum

We first consider $U(N) \times U(N)$ ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ and expand it around the following 1/2-BPS background, which is proportional to unit matrix:

$$\begin{aligned} Y^1 = Y^2 = Y^3 = 0, \quad Y^4 = v e^{-i \frac{\mu t}{2}} \cdot \mathbf{1}, \\ A_0^{(1)} = A_0^{(2)} = 0, \quad A_1^{(1)} = A_1^{(2)} = 0, \\ A_2^{(1)} = A_2^{(2)} = \frac{2\pi v^2 \pm 1 - \cos \theta}{k} \cdot \mathbf{1}, \end{aligned} \quad (4.1)$$

where $v = \sqrt{\frac{\mu k}{2\pi} q}$. We have chosen v to be real by using $U(1)_b$ symmetry. We expand the fields in (2.1) around (4.1) as

$$Y^A \rightarrow \hat{Y}^A + Y^A, \quad A^{(1)} \rightarrow \hat{A}^{(1)} + A^{(1)}, \quad A^{(2)} \rightarrow \hat{A}^{(2)} + A^{(2)}, \quad (4.2)$$

where the hat denotes the background. The limit in which the ABJM theory reduces to SYM is

$$q \rightarrow \infty \quad \text{and} \quad k \rightarrow \infty \quad \text{with} \quad \frac{4\pi\mu q}{k} = \frac{8\pi^2 v^2}{k^2} \equiv g^2 \quad \text{fixed}, \quad (4.3)$$

where g will be identified with the gauge coupling of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ shortly⁸. In this limit, the backgrounds \hat{Y}^4 , $\hat{A}^{(1)}$ and $\hat{A}^{(2)}$ are $\mathcal{O}(k)$. To proceed with the computation, it is convenient to rewrite the gauge fields as follows

$$\begin{aligned} A_m^{(1)} &= A_m + \frac{1}{2k} B_m, \\ A_m^{(2)} &= A_m - \frac{1}{2k} B_m. \end{aligned} \quad (4.4)$$

It turns out that in the limit (4.3) B_m becomes auxiliary fields and can be integrated out while A_m becomes dynamical and will be identified with the gauge field of SYM.

⁸The fact that g^2 is identified with $\frac{8\pi^2 v^2}{k^2}$ instead of $\frac{2\pi v^2}{k}$ as in the BLG case is a matter of notation, and one can go from one to the other by scaling fields by appropriate factors of k .

bosonic part

Ignoring the terms of $\mathcal{O}(k^{-1})$, we obtain

$$\begin{aligned} \int dt \frac{d\Omega}{\mu^2} \text{Tr} \Bigg[& -|D'_a Y^{A'}|^2 - \frac{\mu^2}{4} Y^{A'} Y_{A'}^\dagger + |D'_0 Y^4 + \frac{i}{k} \hat{Y}^4 B_0|^2 - \frac{\mu}{2k} (\hat{Y}^4 Y_4^\dagger + \hat{Y}_4^\dagger Y^4) B_0 \\ & - |D'_1 Y^4 + \frac{i}{k} \hat{Y}^4 B_1|^2 - |D'_2 Y^4 + \frac{i}{k} \hat{Y}^4 B_2|^2 - \frac{\mu^2}{4} Y^4 Y_4^\dagger + \frac{1}{2\pi} (B_0 F_{12} + B_1 F_{20} + B_2 F_{01}) \\ & + \frac{4\pi^2}{k^2} |\hat{Y}^4|^2 \left([Y_{A'}^\dagger, Y^{B'}][Y^{A'}, Y_{B'}^\dagger] + [Y^{A'}, Y^{B'}][Y_{A'}^\dagger, Y_{B'}^\dagger] \right) + \frac{8\pi^2}{k^2} |\hat{Y}^4|^2 [\phi, Y^{A'}][\phi, Y_{A'}^\dagger] \Bigg], \end{aligned} \quad (4.5)$$

where $D'_a = \nabla_a + i[A_a, \cdot]$. Integrating out B_a and rewriting $Y^{A'}$ ($A' = 1, 2, 3$) and Y^4 as

$$\begin{aligned} Y^{A'} &= \frac{1}{\sqrt{2}g} X^{A'4}, \\ Y_{A'}^\dagger &= \frac{1}{\sqrt{2}g} X_{A'4} = \frac{1}{\sqrt{2}g} \cdot \frac{1}{2} \epsilon_{A'B'C'} X^{B'C'}, \\ Y^4 &= \frac{e^{-i\frac{\mu t}{2}}}{\sqrt{2}g} (\phi + i\rho), \end{aligned} \quad (4.6)$$

we finally get

$$\begin{aligned} \frac{1}{g^2} \int dt \frac{d\Omega}{\mu^2} \text{Tr} \Bigg[& -\frac{1}{2} D'_m \phi D'^m \phi - \frac{1}{2} (F_{12} - \mu\phi)^2 + \frac{1}{2} (F_{01})^2 + \frac{1}{2} (F_{20})^2 \\ & - \frac{1}{2} D'_m X_{AB} D'^m X^{AB} - \frac{\mu^2}{8} X_{AB} X^{AB} + \frac{1}{4} [X_{AB}, X_{CD}][X^{AB}, X^{CD}] + \frac{1}{2} [\phi, X_{AB}][\phi, X^{AB}] \Bigg]. \end{aligned} \quad (4.7)$$

To obtain this expression, we have integrated by parts and used Bianchi identity $\epsilon^{abc} D'_a F_{bc} = 0$. The action (4.7) is invariant under $U(N)$ gauge transformation, where the scalar fields ϕ and X_{AB} transform as the adjoint representation of $U(N)$ and D'_m is the adjoint covariant derivative with the gauge field A_m , and also has global $SU(4)$ symmetry. This theory is nothing but (the bosonic part of) $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$.

fermionic part

The details of the fermionic part of $\mathcal{N} = 8$ SYM action are also reproduced by this procedure. The fermionic part of ABJM action has two set of terms: the kinetic term as well as the quartic interaction term involving the fermions and bosons. It turns out from

(4.4) that the effect of the Higgsing procedure on the covariant derivative for the fermions is simply to drop the B_m field in the covariant derivative of ABJM action

$$D_m \psi_A \rightarrow D'_m \psi_A = \nabla_m \psi_A + i[A_m, \psi_A], \quad (4.8)$$

Then the kinetic term of ABJM theory becomes

$$\text{Tr} \left(i \psi^{\dagger A} \gamma^m D'_m \psi_A \right). \quad (4.9)$$

Note that ψ_A here is the fermion field of the SYM and becomes adjoint field in $U(N)$. We now come to the quartic terms, the last two lines in (2.1). By the Higgsing those terms reduce to

$$\begin{aligned} \text{Tr} \left(2ie^{i\frac{\mu t}{2}} \psi^{\dagger 4} [X^{4A'}, \psi_{A'}] - 2ie^{-i\frac{\mu t}{2}} \psi_4 [X_{4A'}, \psi^{\dagger A'}] + i\psi^{\dagger A'} [\phi, \psi_{A'}] - i\psi^{\dagger 4} [\phi, \psi_4] \right. \\ \left. - ie^{-i\frac{\mu t}{2}} \psi^{\dagger A'} [X_{A'B'}, \psi^{\dagger B'}] + ie^{i\frac{\mu t}{2}} \psi_{A'} [X^{A'B'}, \psi_{B'}] \right), \end{aligned} \quad (4.10)$$

where X^{AB} are defined in (4.6).

In what follows, we see that these two, (4.9) and (4.10), can be rewritten in $SU(4)$ symmetric form and are indeed the fermionic part of $\mathcal{N} = 8$ SYM. First we absorb the time-dependence appearing in (4.10) by the following redefinition

$$\begin{aligned} \psi_{A'} &\rightarrow e^{-i\frac{\mu t}{4}} \psi_{A'}, \\ \psi_4 &\rightarrow e^{i\frac{\mu t}{4}} \psi_4. \end{aligned} \quad (4.11)$$

By this, the kinetic term yields mass terms

$$\text{Tr} \left(i \psi^{\dagger A} \gamma^m D'_m \psi_A \right) \rightarrow \text{Tr} \left(i \psi^{\dagger A} \gamma^m D'_m \psi_A + \frac{\mu}{4} \psi^{\dagger A'} \gamma^0 \psi_{A'} - \frac{\mu}{4} \psi^{\dagger 4} \gamma^0 \psi_4 \right). \quad (4.12)$$

Next, in order to see the $SU(4)$ invariance of the action, we regard ψ_4 ($\psi^{\dagger 4}$) which transforms as the forth-component of $\mathbf{4}$ ($\bar{\mathbf{4}}$) of $SU(4)$ in ABJM theory as the field which transforms as the forth-component of $\bar{\mathbf{4}}$ ($\mathbf{4}$). Namely, we interchange ψ_4 and $\psi^{\dagger 4}$,

$$\psi_4 \leftrightarrow \psi^{\dagger 4}. \quad (4.13)$$

The reason of this interchange is explained below. Then (4.10) and (4.12) are rewritten in $SU(4)$ symmetric form as

$$\text{Tr} \left(i \psi^{\dagger A} \gamma^m D'_m \psi_A + \frac{\mu}{4} \psi^{\dagger A} \gamma^0 \psi_A + i \psi^{\dagger A} [\phi, \psi_A] - i \psi^{\dagger A} [X_{AB}, \psi^{\dagger B}] + i \psi_A [X^{AB}, \psi_B] \right) \quad (4.14)$$

The precise correspondence with the form of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ given in appendix C can be seen by performing the following replacements: $\mu \rightarrow -\mu$, $\phi \rightarrow -\phi$, $\psi_A \rightarrow \gamma^0 \hat{\psi}_A^\dagger$ and $\psi^{\dagger A} \rightarrow \gamma^0 \hat{\psi}^A$, where $\hat{\psi}^A$ and $\hat{\psi}_A^\dagger$ are fermions of $\mathcal{N} = 8$ SYM.

The fermions of ABJM theory ψ_A and $\psi^{\dagger A}$ transform as $\mathbf{4}_1$ and $\bar{\mathbf{4}}_{-1}$ under $SU(4) \times U(1)_b$, respectively. By the Higgsing mechanism, $SU(4)$ is broken into $SU(3) \times U(1)$, and thus ψ_A and $\psi^{\dagger A}$ are split into $\mathbf{3}_{1/2} \oplus \mathbf{1}_{3/2}$ and $\bar{\mathbf{3}}_{-1/2} \oplus \mathbf{1}_{-3/2}$, respectively. On the other hand, the fermions of $\mathcal{N} = 8$ SYM are $\mathbf{4}$ and $\bar{\mathbf{4}}$ of $SU(4)$ and not charged under $U(1)_b$ since they are adjoint fields. By decomposing $SU(4)$ into $SU(3) \times U(1)$, $\hat{\psi}_A^\dagger$ and $\hat{\psi}^A$ are split into $\mathbf{3}_{1/2} \oplus \mathbf{1}_{-3/2}$ and $\bar{\mathbf{3}}_{-1/2} \oplus \mathbf{1}_{3/2}$, respectively. To identify the fermions of the ABJM theory with those of $\mathcal{N} = 8$ SYM, we have to set $\psi_{A'} = \hat{\psi}_{A'}^\dagger$ and $\psi_4 = \hat{\psi}^4$ essentially. This is what we have done in the above. (See details in appendix D).

Note that the scalar field ρ , which is the fluctuation of Y^4 , is completely decoupled from the theory since in the limit (4.3) ρ becomes a compact scalar with period $\rho \sim \rho + g^2$, which can be seen from the identification of scalars (3.11) with (4.1), (4.2), (4.3) and (4.6). Note also the difference of the action of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ from that on the flat space. For instance, the scalar field ϕ has the different mass from that of other scalars and the coupling with F_{12} and so there is no $SO(7)$ global symmetry among scalar fields unlike $\mathcal{N} = 8$ SYM on $\mathbf{R}^{1,2}$ where there is no such difference among scalar fields and the $SO(7)$ global symmetry exists. From the perspective of the Higgsing, the scalar field ϕ is coming from the fluctuation around the 1/2-BPS solution (3.5) of Y^4 as (4.6) and the difference from other scalars is coming from the time-dependence of the background around which we expanded ABJM theory on $\mathbf{R} \times \mathbf{S}^2$. This time-dependence is also the source of the mass term of the fermions in the SYM. Now, $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ can also be obtained from the dimensional reduction of $\mathcal{N} = 4$ SYM on $\mathbf{R} \times \mathbf{S}^3(\mathbb{Z}_n)$ onto $\mathbf{R} \times \mathbf{S}^2$, where \mathbf{S}^3 is viewed as \mathbf{S}^1 fiber over \mathbf{S}^2 [29]. It is interesting to note the different origin of the scalar field ϕ and the mass terms from this viewpoint. In this construction, the scalar field ϕ in $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ originates from the gauge field along the fiber direction in $\mathcal{N} = 4$ SYM on $\mathbf{R} \times \mathbf{S}^3(\mathbb{Z}_n)$ and the mass term of the scalar ϕ and that of the fermions from the difference of the spin connection of \mathbf{S}^3 and \mathbf{S}^2 .

One can also carry out the higgsing procedure directly at the level of the super-

symmetry transformations of ABJM theory and show that it reduces to a subset of the full supersymmetry transformations of the SYM⁹. The supersymmetry transformation of ABJM theory (2.4) reduces to that of $\mathcal{N} = 8$ SYM (C.2) by

$$\frac{i}{\sqrt{2}}e^{-i\mu t/4}\xi_{4B'}^{(+)} = \varepsilon_{B'}^\dagger, \quad \frac{i}{\sqrt{2}}e^{i\mu t/4}\xi^{(+)}_{4B'} = -\varepsilon^{B'}, \quad (4.15)$$

with $\varepsilon^4, \varepsilon_4^\dagger = 0$. This means that the enhanced supersymmetry is given by $\varepsilon^4, \varepsilon_4^\dagger$. We will now briefly comment on the symmetry enhancement that happens during the Higgsing process.

While $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ as well as on flat space preserves sixteen supersymmetries, the half-BPS solution of ABJM theory, around which the Higgsing takes place, preserves only twelve supersymmetries. Therefore the Higgsing procedure is accompanied with an enhancement of supersymmetry as well as an enhancement of the associated R-symmetry. This is different from the case of higgsing in the BLG theory, where there is no enhancement of symmetry, since the vacuum of the BLG theory preserves sixteen supersymmetries to begin with.

There is a simple way to understand how this enhancement happens during the process of Higgsing. The effect of the Higgsing can be summarized by some “effective higgsing rules”, as was done for the BLG case [37]. In particular, under the Higgsing procedure, the bi-fundamental covariant derivative action on fields $Y^{A'}$, $Y_{A'}^\dagger$ ($A' = 1, 2, 3$) ($D_m Y^{A'} = \partial_m Y^{A'} + iA_m^{(1)} Y^{A'} - iY^{A'} A_m^{(2)}$) is replaced by an adjoint covariant derivative: ($D'_m Y^{A'} = \partial_m Y^{A'} + i[A_m, Y^{A'}]$). This is true for the covariant derivative of the fermions as well. The solution around which the Higgsing is done preserves only $SU(3) \times U(1)$ of the full global symmetry $SU(4) \times U(1)_b$ of ABJM theory. The conserved currents associated with these symmetries are gauge invariant observables constructed of the $Y^{A'}$ and the $Y_{A'}^\dagger$ and take the form:

$$J_{B'm}^{A'} = \text{Tr}(Y^{A'} D_m Y_{B'}^\dagger) \quad (4.16)$$

The conserved currents associated to the $SO(6)$ symmetry of the SYM would be :

$$j_{B'm}^{A'} = \text{Tr}(Y^{A'} D'_m Y_{B'}^\dagger); \quad \hat{j}_m^{A'B'} = \text{Tr}(Y^{[A'} D'_m Y^{B']}); \quad \hat{j}_{A'B'm}^\dagger = \text{Tr}(Y_{[A'}^\dagger D'_m Y_{B']}^\dagger) \quad (4.17)$$

⁹ In [36], the BPS equations of ABJM theory on flat space was shown to reduce to the BPS equations of SYM under Higgsing.

The additional currents which arise in the SYM limit descend from operators which were not gauge invariant observables in ABJM theory. They become gauge invariant, after Higgsing, under the gauge transformations of the reduced gauge group. This discussion carries over to the enhancement of supercurrents as well.

4.2 $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ around nontrivial vacua

We can also obtain $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ expanded around a nontrivial vacuum, which is presented in appendix C. To see this, let us choose a more general 1/2-BPS background, which is diagonal but not proportional to unit matrix;

$$\begin{aligned} Y^1 = Y^2 = Y^3 = 0, \quad Y^4 = \text{diag}(v_1, v_2, \dots, v_N) e^{-i\frac{\mu t}{2}}, \\ A_0^{(1)} = A_0^{(2)} = 0, \quad A_1^{(1)} = A_1^{(2)} = 0, \\ A_2^{(1)} = A_2^{(2)} = \frac{2\pi}{k} |Y^4|^2 \frac{\pm 1 - \cos \theta}{\sin \theta}. \end{aligned} \quad (4.18)$$

Here

$$v_i = \sqrt{\frac{\mu k}{2\pi} (q + q_i)}, \quad (4.19)$$

where q and q_i are positive half-integers. The theory expanded around such a background is equivalent to the one expanded around (4.1) in which the fluctuation of Y^4 , for instance, is replaced by

$$(Y^4)_{ij} \rightarrow (Y^4)_{ij} + \delta_{ij} (v_i - v) e^{-i\frac{\mu t}{2}}. \quad (4.20)$$

In the limit (4.3), $v_i - v$ becomes

$$v_i - v \rightarrow \frac{\mu}{\sqrt{2}g} q_i \quad (4.21)$$

and so is regarded as the background of the fluctuation. Under the Higgsing around (4.18), ABJM theory on $\mathbf{R} \times \mathbf{S}^2$, therefore, reduces to $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ expanded around

$$\begin{aligned} \phi = \mu \text{diag}(q_1, q_2, \dots, q_N), \quad X_{AB} = 0, \\ A_0 = 0, \quad A_1 = 0, \quad A_2 = \phi \frac{\pm 1 - \cos \theta}{\sin \theta}. \end{aligned} \quad (4.22)$$

Since the solution (4.18) we expanded the ABJM theory around is also 1/2-BPS as in the previous case, it is expected that (4.22) keeps same amount of supersymmetries as the trivial vacuum of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$. Indeed, as presented in appendix C the configuration (4.22) is a (nontrivial) vacuum of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$.

4.3 $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ around 1/2-BPS solution

It is also possible to obtain $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ expanded around 1/2-BPS solutions by Higgsing ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ about a diagonal 1/4-BPS solution in which Y^A take the form

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ (Y^4)_{ij} &= \delta_{ij} \sum_{p \in \mathbf{Z}_{\geq 0 + \frac{n}{k}}} v_{ip} \sin^p \theta e^{ip\varphi - i(p + \frac{1}{2})\mu t}. \end{aligned} \quad (4.23)$$

In particular, we first take a solution with $n = 0$, namely $p = l \in \mathbf{Z}_{\geq 0}$. The gauge field configuration is also diagonal and each component is given by (3.22) with v_p replaced by v_{il} for each component. In particular, we choose v_{il} as

$$\begin{aligned} v_{i0} &= \sqrt{\frac{\mu k}{2\pi}} (q + q_{i0} + \beta_{i0}), \\ v_{il} &= \frac{e^{i\alpha_{il}}}{c_l} \sqrt{\frac{\mu k}{2\pi}} \beta_{il} \quad (l \geq 1), \end{aligned} \quad (4.24)$$

where q and q_{i0} are positive half-integers and β_{il} are real constants with $\sum_{l \geq 0} \beta_{il} = 0$. c_l is defined in (3.21) and α_{il} are real constants. ABJM theory around this background is the same as the one around the background (4.1) with the fluctuation of Y^4 replaced by

$$(Y^4)_{ij} \rightarrow (Y^4)_{ij} + \delta_{ij} \left(\sum_{l \geq 0} v_{il} \sin^l \theta e^{il\varphi - i(l + \frac{1}{2})\mu t} - v e^{-i\frac{\mu t}{2}} \right). \quad (4.25)$$

Then, under the limit in which

$$q \rightarrow \infty, \quad k \rightarrow \infty \quad \text{and} \quad \beta_{il} \rightarrow 0 \quad \text{with} \quad \frac{4\pi\mu q}{k} \equiv g^2 \quad \text{and} \quad v_{il} (\sim \sqrt{k\beta_{il}}) \quad \text{fixed.} \quad (4.26)$$

the second term in the right-hand side in (4.25) becomes

$$\sum_{l \geq 0} v_{il} \sin^l \theta e^{il\varphi - i(l + \frac{1}{2})\mu t} - v e^{-i\frac{\mu t}{2}}$$

$$\rightarrow \frac{\mu}{\sqrt{2}g} q_{i0} e^{-i\frac{\mu t}{2}} + \sum_{l \geq 1} v_{il} \sin^l \theta e^{il\varphi - i(l+\frac{1}{2})\mu t}, \quad (4.27)$$

So, the theory we finally get is $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ around

$$\begin{aligned} \phi_{ij} &= \delta_{ij} \left(\mu q_{i0} + \frac{g}{\sqrt{2}} \sum_{l \geq 1} \sin^l \theta (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \right), \\ X_{AB} &= 0, \\ (A_0)_{ij} &= \delta_{ij} \frac{g}{\sqrt{2}} \sum_{l \geq 1} l (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-l+r}{r} (\mp 1 + \cos^{2r+1} \theta), \\ A_1 &= 0, \\ (A_2)_{ij} &= \delta_{ij} \left[\mu q_{i0} \frac{\pm 1 - \cos \theta}{\sin \theta} \right. \\ &\quad + \frac{g}{\sqrt{2}} \sum_{l \geq 1} (l+1) (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \\ &\quad \left. \times \frac{1}{\sin \theta} \sum_{r=0}^{\infty} \frac{1}{2r+1} \binom{-l+r-1}{r} (\pm 1 - \cos^{2r+1} \theta) \right]. \end{aligned} \quad (4.28)$$

The field strength for the above gauge field configuration is give by

$$\begin{aligned} (F_{01})_{ij} &= \delta_{ij} \frac{\mu g}{\sqrt{2}} \sum_{l \geq 1} l \sin^{l-1} \theta (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}), \\ (F_{02})_{ij} &= \delta_{ij} \frac{\mu g i}{\sqrt{2}} \sum_{l \geq 1} l \cos \theta \sin^{l-1} \theta (v_{il} e^{il(\varphi - \mu t)} - \text{c.c.}), \\ (F_{12})_{ij} &= \delta_{ij} \left(\mu^2 q_{i0} + \frac{\mu g}{\sqrt{2}} \sum_{l \geq 1} (l+1) \sin^l \theta (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \right). \end{aligned} \quad (4.29)$$

It turns out from the Killing spinor equation $\delta \hat{\psi}^A = 0$ of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ given in appendix C that the field configuration (4.28) is a 1/2-BPS solution of the SYM¹⁰.

One can also carry out the Higgsing to a solution with $n \neq 0$ in (4.23). In the same manner as before, we take v_{ip} ($p \in \mathbf{Z}_{\geq 0} + \frac{n}{k}$) as

$$v_{i\frac{n}{k}} = \frac{1}{c_{\frac{n}{k}}} \sqrt{\frac{\mu k}{2\pi} (q + q_{i\frac{n}{k}} + \beta_{i\frac{n}{k}})},$$

¹⁰ As discussed in [35] (also in [30]), the plane wave (BMN) matrix model can be regarded as a matrix regularization of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$. So, there should be 1/2-BPS solutions in the plane wave matrix model corresponding to (4.28). Indeed one of 1/2-BPS solutions in the plane wave matrix model studied in [38] seems to correspond to (4.28).

$$v_{ip} = \frac{e^{i\alpha_{ip}}}{c_p} \sqrt{\frac{\mu k}{2\pi}} \beta_{ip} \quad \left(p \in \mathbf{Z}_{\geq 1} + \frac{n}{k} \right), \quad (4.30)$$

and take the limit in which

$$q \rightarrow \infty, \quad k \rightarrow \infty \quad \text{and} \quad \beta_{ip} \rightarrow 0 \quad \text{with} \quad \frac{4\pi\mu q}{k} \equiv g^2 \quad \text{and} \quad v_{ip} (\sim \sqrt{k\beta_{ip}}) \quad \text{fixed.} \quad (4.31)$$

The effect of $n(\neq 0)$ results in extra terms being added to the previous result. For instance, in the $k \rightarrow \infty$ limit, $\sin^{\frac{n}{k}} \theta$ is approximated as $\sin^{\frac{n}{k}} \theta \rightarrow 1 + \frac{n}{k} \ln \sin \theta + \mathcal{O}((\frac{n}{k})^2)$, which is valid except at $\theta = 0$ and π , and $v_{i(l+\frac{n}{k})}$ can be regarded as v_{il} in (4.24) times a constant:

$$v_{i(l+\frac{n}{k})} \rightarrow v_{il} \times \left(1 + \frac{n}{k} \ln 2 + \mathcal{O}\left(\left(\frac{n}{k}\right)^2\right) \right). \quad (4.32)$$

Then, (4.23) with $n \neq 0$ reduces to, except at $\theta = 0$ and π ,

$$\begin{aligned} & \sum_{p \in \mathbf{Z}_{\geq 0} + \frac{n}{k}} v_{ip} \sin^p \theta e^{ip\varphi - i(p+\frac{1}{2})\mu t} \\ & \rightarrow v e^{-i\frac{\mu t}{2}} + \left[\frac{g}{2\sqrt{2}\pi} n \left(\ln \frac{\sin \theta}{2} + i(\varphi - \mu t) \right) + \frac{\mu}{\sqrt{2}g} q_{i0} + \sum_{p \geq 1} v_p \sin^p \theta e^{ip(\varphi - \mu t)} \right] e^{-i\frac{\mu t}{2}}. \end{aligned} \quad (4.33)$$

The second term is the new term arising due to the nonzero n . One can easily carry out the same calculations for the gauge field configurations. Thus the configurations in the SYM obtained from the 1/4-BPS solutions with nonzero n of ABJM theory via the Higgsing are

$$\begin{aligned} \phi_{ij} &= \delta_{ij} \left(\mu q_{i0} + \frac{ng^2}{2\pi} \ln \frac{\sin \theta}{2} + \frac{g}{\sqrt{2}} \sum_{l \geq 1} \sin^l \theta (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \right), \\ X_{AB} &= 0, \\ (A_0)_{ij} &= \delta_{ij} \left[-\frac{\mu ng^2}{2\pi} \ln \tan \frac{\theta}{2} \right. \\ & \quad \left. + \frac{g}{\sqrt{2}} \sum_{l \geq 1} l (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \sum_{r=0}^{l-1} \frac{1}{2r+1} \binom{-l+r}{r} (\mp 1 + \cos^{2r+1} \theta) \right], \\ A_1 &= 0, \\ (A_2)_{ij} &= \delta_{ij} \left[\mu q_{i0} \frac{\pm 1 - \cos \theta}{\sin \theta} + \frac{ng^2}{2\pi} \left(\frac{1 - \cos \theta}{\sin \theta} \ln \sin \frac{\theta}{2} - \frac{1 + \cos \theta}{\sin \theta} \ln \cos \frac{\theta}{2} \right) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{g}{\sqrt{2}} \sum_{l \geq 1} (l+1) (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \\
& \times \frac{1}{\sin \theta} \sum_{r=0}^l \frac{1}{2r+1} \binom{-l+r-1}{r} (\pm 1 - \cos^{2r+1} \theta) \Big]. \quad (4.34)
\end{aligned}$$

The field strength for the above gauge field configuration is give by

$$\begin{aligned}
(F_{01})_{ij} &= \delta_{ij} \left(\frac{\mu n g^2}{2\pi} \frac{1}{\sin \theta} + \frac{\mu g}{\sqrt{2}} \sum_{l \geq 1} l \sin^{l-1} \theta (v_{il} e^{ip(\varphi - \mu t)} + \text{c.c.}) \right), \\
(F_{02})_{ij} &= \delta_{ij} \frac{\mu g i}{\sqrt{2}} \sum_{l \geq 1} l \cos \theta \sin^{l-1} \theta (v_{il} e^{il(\varphi - \mu t)} - \text{c.c.}), \\
(F_{12})_{ij} &= \delta_{ij} \left[\mu^2 q_{i0} + \frac{\mu n g^2}{2\pi} \left(1 + \ln \frac{\sin \theta}{2} \right) + \frac{\mu g}{\sqrt{2}} \sum_{l \geq 1} (l+1) \sin^l \theta (v_{il} e^{il(\varphi - \mu t)} + \text{c.c.}) \right]. \quad (4.35)
\end{aligned}$$

Note that the terms proportional to n appearing in F_{01} and A_0 can be regarded as analogue on $\mathbf{R} \times \mathbf{S}^2$ of the Callan-Maldacena solution on flat space [39], which describes a bound state of fundamental strings and D2-branes. This part in the solution represents n fundamental strings attaching D2-branes on the north pole ($\theta = 0$) and the south pole ($\theta = \pi$). The behavior around them indeed matches with the solution [40]. On the other hand, the expressions for F_{12} and A_2 are specific to the analysis on $\mathbf{R} \times \mathbf{S}^2$. F_{12} is singular at $\theta = 0$ and $\theta = \pi$ but A_2 is not. Note also that the integral of the new term in F_{12} over \mathbf{S}^2 vanishes as well as that of the terms of $l \geq 1$, so the flux quantization condition is just $\frac{1}{2\pi\mu^2} \int_{\mathbf{S}^2} (F_{12})_{ii} = 2q_{i0} \in \mathbf{Z}$, which is consistent with that in ABJM theory.

5 Summary and Discussion

In summary, we have solved BPS equations of ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ for diagonal configurations and shown that “Higgsing” the ABJM theory around the 1/2-BPS solution leads to $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$. The BPS solutions we found, in general, have nonzero angular momentum along φ direction and the non-trivial fluxes, not only F_{12} but also F_{01} and F_{02} . Higgsing around the 1/2-BPS solution where the scalar field vev is proportional to the identity gives rise to $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ expanded around the trivial vacuum while higgsing around 1/2-BPS solutions which are diagonal but not proportional to the identity leads to the SYM expanded around a non-trivial vacuum. If we Higgs around

a 1/4-BPS configuration, then we end up getting the SYM expanded around a 1/2-BPS solution. In fact, higgsing around various solutions of ABJM theory should reproduce the SYM expanded around its various solutions.

Since the ABJM on $\mathbf{R} \times \mathbf{S}^2$ is dual to M-theory on global AdS_4 , it is worth asking what the duals of the BPS solutions, we find in this paper, are. In [41], Nishioka and Takayanagi solve the BPS equations explicitly in the bulk and construct a class of dual giant graviton solutions in M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$. In particular, they find a spinning dual giant graviton configuration. The spinning dual giant graviton is a M2-brane expanding into AdS_4 , which rotates along the fiber coordinate of the S^7 (S^7 being the fibration of S^1 over \mathbb{CP}^3) and spins along the azimuthal direction of $S^2 \subset AdS_4$. This spinning dual giant graviton has a non-trivial profile along the AdS_4 and has been called the “giant torus”. These solutions should be dual to the class of solutions we construct in this paper with nonzero P_φ and J_4 corresponding to the nonzero spin and the angular momentum, respectively, in the bulk.

In a forthcoming paper [42], we will classify the space of solutions on the bulk side, which includes the giant torus solution, in terms of intersections of holomorphic surfaces with the target space, following [43, 44] and then using the methods given in [45–47] we will compare and match with a similar classification on the space of boundary solutions presented here.

Acknowledgment: We would like to thank collectively Rajesh Gopakumar, Hikaru Kawai, Seok Kim, Tsunehide Kuroki, Suvrat Raju, Nemani Suryanarayana for useful discussions. The work of S.S. is supported in part by the JSPS Research Fellowship for Young Scientists. SY is supported by National Research Foundation of Korea (NRF) grant No. 2010-0007512, and No. 2009-0076297.

A Conventions

In this paper, we consider the ABJM theory on $\mathbf{R} \times \mathbf{S}^2$ endowed with the metric

$$ds^2 = -dt^2 + \frac{1}{\mu^2} (d\theta^2 + \sin^2 \theta d\varphi^2), \quad (\text{A.1})$$

where μ^{-1} is the radius of S^2 . We take the local Lorentz frame as

$$e^0 = dt, \quad e^1 = \frac{1}{\mu} d\theta, \quad e^2 = \frac{1}{\mu} \sin \theta d\varphi. \quad (\text{A.2})$$

Then the spin connection is calculated as

$$\omega_{12} = -\cos \theta d\varphi, \quad \text{others} = 0. \quad (\text{A.3})$$

We take $SO(1, 2)$ gamma matrices, which satisfy $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$, as

$$\gamma^0 = i\sigma_y, \quad \gamma^1 = \sigma_x, \quad \gamma^2 = \sigma_z, \quad (\text{A.4})$$

where $\sigma_{x,y,z}$ are Pauli matrices. Note that

$$\gamma^a \gamma^b = \eta^{ab} + \epsilon^{ab}{}_c \gamma^c, \quad (\text{A.5})$$

where ϵ^{abc} is the completely antisymmetric tensor satisfying $\epsilon^{012} = 1$. In this representation, spinors are real. Let spinors and the gamma matrices have the following index structure: $\psi_\alpha, (\gamma^a)_\alpha{}^\beta$. We raise and lower the indices by the antisymmetric tensor $\epsilon^{\alpha\beta}$ and $\epsilon_{\alpha\beta}$ satisfying $\epsilon^{12} = -\epsilon_{12} = 1$ as $\psi^\alpha \equiv \epsilon^{\alpha\beta} \psi_\beta$ ($\psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta$), $(\gamma^a)_{\alpha\beta} \equiv \epsilon_{\beta\beta'} (\gamma^a)_\alpha{}^{\beta'}$ and $(\gamma^a)^{\alpha\beta} \equiv \epsilon^{\alpha\alpha'} (\gamma^a)_{\alpha'}{}^\beta$. The gamma matrices with two upper indices and two lower indices are symmetric: $(\gamma^a)^{\alpha\beta} = (\gamma^a)^{\beta\alpha}$ and $(\gamma^a)_{\alpha\beta} = (\gamma^a)_{\beta\alpha}$. We abbreviate the spinor indices for the following contractions:

$$\begin{aligned} \psi\chi &\equiv \psi^\alpha \chi_\alpha = \chi\psi, \\ \psi\gamma^{a_1} \cdots \gamma^{a_k} \chi &\equiv \psi^\alpha (\gamma^{a_1} \cdots \gamma^{a_k})_\alpha{}^\beta \chi_\beta \end{aligned} \quad (\text{A.6})$$

B BPS solutions

In this appendix, we summarize the BPS solutions of $U(1) \times U(1)$ ABJM theory ($k > 2$) with respect to the cases in which $\eta_{AB}^{(+)}$ take

$$\begin{aligned} \text{(i)} : \quad & \eta_{14}^{(+)} \neq 0 \text{ and others} = 0, \\ \text{(ii)} : \quad & \eta_{14}^{(+)}, \eta_{24}^{(+)} \neq 0 \text{ and others} = 0, \\ \text{(iii)} : \quad & \eta_{14}^{(+)}, \eta_{24}^{(+)}, \eta_{34}^{(+)} \neq 0 \text{ and others} = 0. \end{aligned} \quad (\text{B.1})$$

	without (B.2) and (B.3)	with (B.2) and (B.3)
(i)	4	2
(ii)	8	4
(iii)	12	6

Table 1: The number of supersymmetries for each BPS condition in ABJM on $\mathbf{R} \times \mathbf{S}^2$ ($k > 2$): (i) $\eta_{14}^{(+)} \neq 0$ and $\eta_{24}^{(+)} = \eta_{34}^{(+)} = 0$, (ii) $\eta_{14}^{(+)}, \eta_{24}^{(+)} \neq 0$ and $\eta_{34}^{(+)} = 0$, and (iii) $\eta_{14}^{(+)}, \eta_{24}^{(+)}, \eta_{34}^{(+)} \neq 0$.

Note that $\eta_{AB}^{(-)} = -\frac{1}{2}\epsilon_{ABCD}(\eta_{CD}^{(+)})^*$. The other cases are essentially the same with one of these cases (for instance, the case in which $\eta_{12}^{(+)} \neq 0$ and others = 0 is equivalent to the case (i).). For nonzero constant spinors, we can further impose the following projection

$$i\gamma^0\eta_{A'4}^{(+)} = s_{A'}\eta_{A'4}^{(+)}, \quad (\text{B.2})$$

where $s_{A'} = \pm 1$. The projection for $\eta_{AB}^{(-)}$ is given by

$$i\gamma^0\eta_{A'B'}^{(-)} = s'_{A'B'}\eta_{A'B'}^{(-)}, \quad (\text{B.3})$$

with $s'_{12} = s'_{21} = -s_3, s'_{13} = s'_{31} = -s_2, s'_{23} = s'_{32} = -s_1$. The number of supersymmetries preserved for each case in (B.1) with and without (B.2) and (B.3) is summarized in Table 1. From (3.1) one can easily get the BPS configurations of scalar fields for each case and then those of gauge fields from (2.3). Below we show the BPS solutions of scalar fields for each case.

In the case (i) with (B.2) and (B.3), (3.1) reduces to the following equations:

$$\begin{aligned} \partial_t Y^{\bar{A}} + i\frac{\mu}{2}Y^{\bar{A}} + s_1\mu\partial_\varphi Y^{\bar{A}} &= 0, \\ \partial_\theta Y^{\bar{A}} + is_1\cot\theta\partial_\varphi Y^{\bar{A}} &= 0, \\ \partial_t Y^{\underline{A}} - i\frac{\mu}{2}Y^{\underline{A}} + s_1\mu\partial_\varphi Y^{\underline{A}} &= 0, \\ \partial_\theta Y^{\underline{A}} - is_1\cot\theta\partial_\varphi Y^{\underline{A}} &= 0, \end{aligned} \quad (\text{B.4})$$

where $\bar{A} = 1, 4$ and $\underline{A} = 2, 3$. These are easily solved as

$$\begin{aligned} Y^{\bar{A}} &= \sum_{p \in \mathbf{Z}_{\geq 0}} v_p^{\bar{A}} \sin^p \theta e^{ip(s_1\varphi - t) - i\frac{\mu t}{2}}, \\ Y^{\underline{A}} &= \sum_{p \in \mathbf{Z}_{\geq 0}} v_p^{\underline{A}} \sin^p \theta e^{-ip(s_1\varphi - t) + i\frac{\mu t}{2}}, \end{aligned} \quad (\text{B.5})$$

where $v_p^{\bar{A}}$ and v_p^A are arbitrary constants. Note that if $Y^{\bar{A}} = 0$ ($v_p^{\bar{A}} = 0$) then p of $v_p^{\bar{A}}$ can take values in $\mathbf{Z}_{\geq 0} + \frac{n}{k}$, where n is an integer with $0 \leq n < k$, because of the identification (3.11):

$$\begin{aligned} Y^{\bar{A}} &= \sum_{p \in \mathbf{Z}_{\geq 0} + \frac{n}{k}} v_p^{\bar{A}} \sin^p \theta e^{ip(s_1 \varphi - t) - i \frac{\mu t}{2}}, \\ Y^{\bar{A}} &= 0. \end{aligned} \quad (\text{B.6})$$

Without (B.2) and (B.3), the BPS equation becomes (B.4) with the coefficient of s_1 being zero, so that the corresponding BPS solution is $p = 0$ solution in (B.5).

In the case (ii) with (B.2) and (B.3), the BPS solution is given, only when $s_1 = s_2$, by

$$\begin{aligned} Y^1 &= Y^2 = 0, \\ Y^4 &= \sum_{p \in \mathbf{Z}_{\geq 0}} v_p^4 \sin^p \theta e^{ip(s_1 \varphi - t) - i \frac{\mu t}{2}}, \\ Y^3 &= \sum_{p \in \mathbf{Z}_{\geq 0}} v_p^3 \sin^p \theta e^{-ip(s_1 \varphi - t) + i \frac{\mu t}{2}}. \end{aligned} \quad (\text{B.7})$$

The BPS solution without (B.2) and (B.3) is the solution with $p = 0$ in (B.7).

In the case (iii) with (B.2) and (B.3), the BPS solution is given, only when $s_1 = s_2 = s_3$, by

$$\begin{aligned} Y^1 &= Y^2 = Y^3 = 0, \\ Y^4 &= \sum_{p \in \mathbf{Z}_{\geq 0} + \frac{n}{k}} v_p^4 \sin^p \theta e^{ip(s_1 \varphi - t) - i \frac{\mu t}{2}}, \end{aligned} \quad (\text{B.8})$$

where we have taken into account the identification (3.11), so that p can take an integer of $\mathbf{Z}_{\geq 0} + \frac{n}{k}$. The BPS solution without (B.2) and (B.3) is the solution with $p = 0$ in (B.8).

C $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$

In this appendix, we summarize $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$. The action of $\mathcal{N} = 8$ SYM on $\mathbf{R} \times \mathbf{S}^2$ is given by

$$\begin{aligned} S_{SYM} &= \frac{1}{g_{SYM}^2} \int dt \frac{d\Omega}{\mu^2} \text{Tr} \left(-\frac{1}{4} F^{ab} F_{ab} - \frac{1}{2} D'_a \phi D'^a \phi - \frac{\mu^2}{2} \phi^2 + \mu \phi F_{12} \right. \\ &\quad \left. - \frac{1}{2} D'_a X_{AB} D'^a X^{AB} - \frac{\mu^2}{8} X_{AB} X^{AB} + \frac{1}{2} [\phi, X_{AB}] [\phi, X^{AB}] + \frac{1}{4} [X_{AB}, X_{CD}] [X^{AB}, X^{CD}] \right) \end{aligned}$$

$$\begin{aligned}
& + i\hat{\psi}_A^\dagger \gamma^a D'_a \hat{\psi}^A + \frac{\mu}{4} \hat{\psi}_A^\dagger \gamma^0 \hat{\psi}^A \\
& - i\hat{\psi}_A^\dagger [\phi, \hat{\psi}^A] - i\hat{\psi}_A^\dagger [X^{AB}, \hat{\psi}_B^\dagger] + i\hat{\psi}^A [X_{AB}, \hat{\psi}^B] \Big), \tag{C.1}
\end{aligned}$$

where $D'_a = \nabla_a + i[A_a, \cdot]$. This theory is invariant under the following supersymmetry transformation

$$\begin{aligned}
\delta A^a &= i\varepsilon_A^\dagger \gamma^a \hat{\psi}^A + i\varepsilon^A \gamma^a \hat{\psi}_A^\dagger, \\
\delta \phi &= \varepsilon_A^\dagger \hat{\psi}^A - \varepsilon^A \hat{\psi}_A^\dagger, \\
\delta X^{AB} &= \epsilon^{ABCD} \varepsilon_C^\dagger \hat{\psi}_D - \varepsilon^A \hat{\psi}^B + \varepsilon^B \hat{\psi}^A, \\
\delta \hat{\psi}^A &= -iD'_a \phi \gamma^a \varepsilon^A + \sum_{i=1,2} F_{0i} \gamma^{0i} \varepsilon^A - 2iD'_a X^{AB} \gamma^a \epsilon_B^* \\
&+ (F_{12} - \mu\phi) \gamma^{12} \varepsilon^A + \mu X^{AB} \gamma^{12} \varepsilon_B^* + 2i[\phi, X^{AB}] \varepsilon_B^* + 2i[X^{AB}, X_{BC}] \varepsilon^C \tag{C.2}
\end{aligned}$$

Here ε^A are supersymmetry parameters which are $(1+2)$ -dimensional Majorana spinors in the fundamental representation $(\mathbf{4})$ of $SU(4)$ given by

$$\varepsilon^A = e^{i\frac{\mu t}{4}} e^{-i\frac{\theta}{2}\gamma^2} e^{\frac{\varphi}{2}\gamma^0} \varepsilon_0^A, \tag{C.3}$$

where ε_0^A is a constant spinor. ε_A^* are the complex conjugate of ε^A and transform as the anti-fundamental representation of $SU(4)$.

The vacuum configuration of this theory is determined by the following equations

$$\begin{aligned}
F_{12} - \mu\phi &= 0, \\
D'_1 \phi &= D'_2 \phi = 0. \tag{C.4}
\end{aligned}$$

In the gauge in which ϕ is diagonal and $A_1 = 0$, these equations are solved by introducing two patches on \mathbf{S}^2 as

$$\begin{aligned}
\phi &= \mu \operatorname{diag}(q_1, q_2, \dots, q_N), \\
A_1 &= 0, \\
A_2 &= \frac{1 \pm \cos \theta}{\sin \theta} \phi, \tag{C.5}
\end{aligned}$$

where the upper and lower signs in A_2 correspond to the region I ($0 \leq \theta < \pi$) and the region II ($0 < \theta \leq \pi$), respectively. The gauge field configuration for each diagonal

component is Dirac monopole with magnetic charge q_i . In the overlapping region of the region I and the region II, the configurations on each patch are transformed each other by the transition function

$$V_{\text{I} \rightarrow \text{II}} = \exp \left(i \frac{2}{\mu} \phi \cdot \varphi \right) \quad (\text{C.6})$$

The single-valuedness of the transition function requires q_i to be half-integer: $q_i \in \mathbf{Z}/2$.

D Relation of fermions in ABJM and SYM

In this appendix, we explain in detail the interchange of ψ_4 and $\psi^{\dagger 4}$ (4.13) in the ABJM theory, which is needed for matching the ABJM theory (after the Higgsing) to $\mathcal{N} = 8$ SYM. It is worthwhile to understand this interchange in terms of Clifford algebra representations of $SO(6)$ and $SO(8)$. Let $\bar{\Gamma}^{I'}$ ($I' = 1, 2, \dots, 6$) be gamma matrices of $SO(6)$ satisfying $\{\bar{\Gamma}^{I'}, \bar{\Gamma}^{J'}\} = 2\delta^{I'J'}$ and $\alpha^{A'} = \frac{1}{2}(\bar{\Gamma}^{A'} + i\bar{\Gamma}^{A'+3})$ and $\alpha_{A'}^\dagger = \frac{1}{2}(\bar{\Gamma}^{A'} - i\bar{\Gamma}^{A'+3})$. $\alpha^{A'}$ and $\alpha_{A'}^\dagger$ satisfy $\{\alpha^{A'}, \alpha_{B'}^\dagger\} = \delta_{B'}^{A'}$ and are regarded as annihilation and creation operators of fermions on the Fock vacuum $|\bar{\Omega}\rangle$. Note that the $U(3)$ rotation defined by $\alpha^{A'} \rightarrow (U^*)_{B'}^{A'} \alpha^{B'}$ and $\alpha_{A'}^\dagger \rightarrow U_{A'}^{B'} \alpha_{B'}^\dagger$ is a subgroup of $SO(6)$. The (Dirac) spinor representation of $SO(6)$ is expressed as

$$\mathbf{8} = \{|\bar{\Omega}\rangle, \alpha_{A'}^\dagger |\bar{\Omega}\rangle, \alpha_{A'}^\dagger \alpha_{B'}^\dagger |\bar{\Omega}\rangle, \alpha_{A'}^\dagger \alpha_{B'}^\dagger \alpha_{C'}^\dagger |\bar{\Omega}\rangle\}, \quad (\text{D.1})$$

One can decompose $\mathbf{8}$ in terms of the eigenvalue of the chirality matrix $\bar{\Gamma} = \prod_{I'=1}^6 \bar{\Gamma}^{I'} = \prod_{A=1}^4 (1 - 2\alpha_A^\dagger \alpha^A)$ into two Weyl representations as

$$\mathbf{8} \rightarrow \mathbf{4} + \bar{\mathbf{4}} \quad (\text{D.2})$$

where

$$\begin{aligned} \mathbf{4} &= \left\{ \alpha_{A'}^\dagger |\bar{\Omega}\rangle, \alpha_{A'}^\dagger \alpha_{B'}^\dagger \alpha_{C'}^\dagger |\bar{\Omega}\rangle \right\}, \\ \bar{\mathbf{4}} &= \left\{ |\bar{\Omega}\rangle, \alpha_{A'}^\dagger \alpha_{B'}^\dagger |\bar{\Omega}\rangle \right\}. \end{aligned} \quad (\text{D.3})$$

and $\mathbf{4}$ and $\bar{\mathbf{4}}$ have $\bar{\Gamma} = 1$ and $\bar{\Gamma} = -1$, respectively. We further decompose $\mathbf{4}$ and $\bar{\mathbf{4}}$ of $SU(4)$ into $SU(3) \times U(1)$ where the $U(1)$ charge is specified by $\sum_{A'=1}^3 [\alpha^{A'}, \alpha_{A'}^\dagger]/2$:

$$\mathbf{4} \rightarrow \mathbf{3}_{1/2} + \mathbf{1}_{-3/2},$$

$$\bar{\mathbf{4}} \rightarrow \bar{\mathbf{3}}_{-1/2} + \mathbf{1}_{3/2}. \quad (\text{D.4})$$

Next, let Γ^I ($I = 1, 2, \dots, 8$) be the gamma matrices of $SO(8)$ satisfying $\{\Gamma^I, \Gamma^J\} = 2\delta^{IJ}$ and $\beta^A = \frac{1}{2}(\Gamma^A + i\Gamma^{A+4})$ and $\beta_A^\dagger = \frac{1}{2}(\Gamma^A - i\Gamma^{A+4})$. β^A and β_A^\dagger satisfy $\{\beta^A, \beta_B^\dagger\} = \delta_B^A$ and are regarded as annihilation and creation operators of fermions on Fock vacuum $|\Omega\rangle$. By using the fermion Fock space, the (Dirac) spinor representation of $SO(8)$, $\mathbf{16}$, is given as:

$$\mathbf{16} = \{|\Omega\rangle, \beta_A^\dagger|\Omega\rangle, \beta_A^\dagger\beta_B^\dagger|\Omega\rangle, \beta_A^\dagger\beta_B^\dagger\beta_C^\dagger|\Omega\rangle, \beta_A^\dagger\beta_B^\dagger\beta_C^\dagger\beta_D^\dagger|\Omega\rangle\}. \quad (\text{D.5})$$

In terms of the eigenvalue of the chirality matrix $\Gamma \equiv \prod_{I=1}^8 \Gamma^I = \prod_{A=1}^4 (1 - 2\beta_A^\dagger\beta^A)$, $\mathbf{16}$ is decomposed as

$$\mathbf{16} \rightarrow \mathbf{8}_s + \mathbf{8}_c, \quad (\text{D.6})$$

where

$$\begin{aligned} \mathbf{8}_s &= \left\{ \beta_A^\dagger|\Omega\rangle, \beta_A^\dagger\beta_B^\dagger\beta_C^\dagger|\Omega\rangle \right\}, \\ \mathbf{8}_c &= \left\{ |\Omega\rangle, \beta_A^\dagger\beta_B^\dagger|\Omega\rangle, \beta_A^\dagger\beta_B^\dagger\beta_C^\dagger\beta_D^\dagger|\Omega\rangle \right\}, \end{aligned} \quad (\text{D.7})$$

and $\Gamma = -1$ for $\mathbf{8}_s$ and $\Gamma = 1$ for $\mathbf{8}_c$. We decompose these into $SU(4) \times U(1)$ where the $U(1)$ charge specified by $\sum_{A=1}^4 [\beta^A, \beta_A^\dagger]/2$. In particular, $\mathbf{8}_s$ is decomposed as

$$\mathbf{8}_s \rightarrow \mathbf{4}'_1 + \bar{\mathbf{4}}'_{-1}, \quad (\text{D.8})$$

where

$$\begin{aligned} \mathbf{4}'_1 &= \left\{ \beta_A^\dagger|\Omega\rangle \right\}, \\ \bar{\mathbf{4}}'_{-1} &= \left\{ \beta_A^\dagger\beta_B^\dagger\beta_C^\dagger|\Omega\rangle \right\}. \end{aligned} \quad (\text{D.9})$$

We further decompose $SU(4)$ into $SU(3) \times U(1)$ as before with the $U(1)$ charge specified by $\sum_{A'=1}^3 [\beta^{A'}, \beta_{A'}^\dagger]/2$:

$$\begin{aligned} \mathbf{4}' &\rightarrow \mathbf{3}_{1/2} + \mathbf{1}_{3/2}, \\ \bar{\mathbf{4}}' &\rightarrow \bar{\mathbf{3}}_{-1/2} + \mathbf{1}_{-3/2}. \end{aligned} \quad (\text{D.10})$$

We then see that the two sets, (D.4) and (D.10) are not in one to one correspondence with each other. In particular to identify the fermions of the ABJM theory with the fermions of the SYM (after Higgsing), we must interchange $\mathbf{1}_{3/2} \leftrightarrow \mathbf{1}_{-3/2}$. This corresponds to interchanging $\psi_4 \leftrightarrow \psi^{4\dagger}$ in the ABJM.

References

- [1] J. H. Schwarz, “Superconformal Chern-Simons theories,” JHEP **0411** (2004) 078 [arXiv:hep-th/0411077].
- [2] J. Bagger and N. Lambert, “Modeling Multiple M2’s,” Phys. Rev. D **75** (2007) 045020 [arXiv:hep-th/0611108].
- [3] J. Bagger and N. Lambert, “Gauge symmetry and supersymmetry of multiple M2-branes,” Phys. Rev. D **77** (2008) 065008 [arXiv:0711.0955 [hep-th]].
- [4] J. Bagger and N. Lambert, “Comments on multiple M2-branes,” JHEP **0802** (2008) 105 [arXiv:0712.3738 [hep-th]].
- [5] A. Gustavsson, “Algebraic structures on parallel M2-branes,” Nucl. Phys. B **811** (2009) 66 [arXiv:0709.1260 [hep-th]].
- [6] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” JHEP **0810** (2008) 091 [arXiv:0806.1218 [hep-th]].
- [7] A. Gustavsson and S. J. Rey, “Enhanced N=8 Supersymmetry of ABJM Theory on R^{*8} and $R^{*8}/Z(2)$,” arXiv:0906.3568 [hep-th].
- [8] D. Bashkirov and A. Kapustin, “Supersymmetry enhancement by monopole operators,” JHEP **1105**, 015 (2011) [arXiv:1007.4861 [hep-th]].
- [9] J. Bhattacharya and S. Minwalla, “Superconformal Indices for N = 6 Chern Simons Theories,” JHEP **0901** (2009) 014 [arXiv:0806.3251 [hep-th]].
- [10] Y. Imamura and S. Yokoyama, “N=4 Chern-Simons theories and wrapped M-branes in their gravity duals,” Prog. Theor. Phys. **121** (2009) 915 [arXiv:0812.1331 [hep-th]].
- [11] Y. Imamura, “Monopole operators in N=4 Chern-Simons theories and wrapped M2-branes,” Prog. Theor. Phys. **121** (2009) 1173 [arXiv:0902.4173 [hep-th]].
- [12] S. Kim, “The Complete superconformal index for N=6 Chern-Simons theory,” Nucl. Phys. B **821** (2009) 241 [arXiv:0903.4172 [hep-th]].

- [13] Y. Imamura and S. Yokoyama, “A Monopole Index for N=4 Chern-Simons Theories,” Nucl. Phys. B **827** (2010) 183 [arXiv:0908.0988 [hep-th]].
- [14] K. Hosomichi, K. M. Lee, S. Lee, S. Lee and J. Park, “N=5,6 Superconformal Chern-Simons Theories and M2-branes on Orbifolds,” JHEP **0809** (2008) 002 [arXiv:0806.4977 [hep-th]].
- [15] O. Aharony, O. Bergman and D. L. Jafferis, “Fractional M2-branes,” JHEP **0811** (2008) 043 [arXiv:0807.4924 [hep-th]].
- [16] D. L. Jafferis and A. Tomasiello, “A Simple class of N=3 gauge/gravity duals,” JHEP **0810** (2008) 101 [arXiv:0808.0864 [hep-th]].
- [17] D. Martelli and J. Sparks, “Moduli spaces of Chern-Simons quiver gauge theories and AdS(4)/CFT(3),” Phys. Rev. D **78** (2008) 126005 [arXiv:0808.0912 [hep-th]].
- [18] D. Martelli and J. Sparks, “Notes on toric Sasaki-Einstein seven-manifolds and AdS(4) / CFT(3),” JHEP **0811** (2008) 016 [arXiv:0808.0904 [hep-th]].
- [19] A. Hanany and A. Zaffaroni, “Tilings, Chern-Simons Theories and M2 Branes,” JHEP **0810** (2008) 111 [arXiv:0808.1244 [hep-th]].
- [20] A. Hanany, D. Vegh and A. Zaffaroni, “Brane Tilings and M2 Branes,” JHEP **0903** (2009) 012 [arXiv:0809.1440 [hep-th]].
- [21] S. Franco, A. Hanany, J. Park and D. Rodriguez-Gomez, “Towards M2-brane Theories for Generic Toric Singularities,” JHEP **0812** (2008) 110 [arXiv:0809.3237 [hep-th]].
- [22] S. Franco, I. R. Klebanov and D. Rodriguez-Gomez, “M2-branes on Orbifolds of the Cone over $Q^{*1,1,1}$,” JHEP **0908** (2009) 033 [arXiv:0903.3231 [hep-th]].
- [23] S. Mukhi and C. Papageorgakis, “M2 to D2,” JHEP **0805** (2008) 085 [arXiv:0803.3218 [hep-th]].
- [24] J. Distler, S. Mukhi, C. Papageorgakis and M. Van Raamsdonk, “M2-branes on M-folds,” JHEP **0805** (2008) 038 [arXiv:0804.1256 [hep-th]].

- [25] A. Agarwal and D. Young, “Manifest $SO(N)$ invariance and S-matrices of three-dimensional $N=2,4,8$ SYM,” JHEP **1105**, 100 (2011) [arXiv:1103.0786 [hep-th]].
- [26] A. Agarwal and D. Young, “Hidden Local and Non-local Symmetries of S-matrices of $N=2,4,8$ SYM in $D=2+1$,” arXiv:1109.2792 [hep-th].
- [27] D. Berenstein and J. Park, “The BPS spectrum of monopole operators in ABJM: Towards a field theory description of the giant torus,” JHEP **1006** (2010) 073 [arXiv:0906.3817 [hep-th]].
- [28] D. E. Berenstein, J. M. Maldacena and H. S. Nastase, “Strings in flat space and pp waves from $N=4$ superYang-Mills,” JHEP **0204** (2002) 013 [arXiv:hep-th/0202021].
- [29] H. Lin and J. M. Maldacena, “Fivebranes from gauge theory,” Phys. Rev. D **74** (2006) 084014 [arXiv:hep-th/0509235].
- [30] G. Ishiki, S. Shimasaki, Y. Takayama and A. Tsuchiya, “Embedding of theories with $SU(2|4)$ symmetry into the plane wave matrix model,” JHEP **0611** (2006) 089 [arXiv:hep-th/0610038].
- [31] G. Grignani, L. Griguolo, N. Mori and D. Seminara, “Thermodynamics of theories with sixteen supercharges in non-trivial vacua,” JHEP **0710**, 068 (2007) [arXiv:0707.0052 [hep-th]].
- [32] A. Agarwal and D. Young, “ $SU(2|2)$ for Theories with Sixteen Supercharges at Weak and Strong Coupling,” Phys. Rev. D **82**, 045024 (2010) [arXiv:1003.5547 [hep-th]].
- [33] D. Berenstein and D. Trancanelli, “Three-dimensional $N=6$ SCFT’s and their membrane dynamics,” Phys. Rev. D **78**, 106009 (2008) [arXiv:0808.2503 [hep-th]].
- [34] M. M. Sheikh-Jabbari and J. Simon, “On Half-BPS States of the ABJM Theory,” JHEP **0908** (2009) 073 [arXiv:0904.4605 [hep-th]].
- [35] J. M. Maldacena, M. M. Sheikh-Jabbari and M. Van Raamsdonk, “Transverse five-branes in matrix theory,” JHEP **0301** (2003) 038 [arXiv:hep-th/0211139].

- [36] T. Fujimori, K. Iwasaki, Y. Kobayashi and S. Sasaki, “Classification of BPS Objects in $N = 6$ Chern-Simons Matter Theory,” JHEP **1010** (2010) 002 [arXiv:1007.1588 [hep-th]].
- [37] B. Ezhuthachan, S. Mukhi and C. Papageorgakis, “The Power of the Higgs Mechanism: Higher-Derivative BLG Theories,” JHEP **0904** (2009) 101 [arXiv:0903.0003 [hep-th]].
- [38] J. Hoppe and K. M. Lee, JHEP **0806**, 041 (2008) [arXiv:0712.3616 [hep-th]].
- [39] C. G. Callan and J. M. Maldacena, Nucl. Phys. B **513**, 198 (1998) [arXiv:hep-th/9708147].
- [40] T. Kuroki, A. Miwa and S. Okuda, JHEP **1105**, 011 (2011) [arXiv:1102.3277 [hep-th]].
- [41] T. Nishioka and T. Takayanagi, “Fuzzy Ring from M2-brane Giant Torus,” JHEP **0810** (2008) 082 [arXiv:0808.2691 [hep-th]].
- [42] Bobby Ezhuthachan, Shinji Shimasaki and Shuichi Yokoyama, (Work in progress)
- [43] A. Mikhailov, “Giant gravitons from holomorphic surfaces,” JHEP **0011** (2000) 027 [arXiv:hep-th/0010206].
- [44] S. Kim and K. M. Lee, “1/16-BPS Black Holes and Giant Gravitons in the $AdS(5) \times S^{*5}$ Space,” JHEP **0612** (2006) 077 [arXiv:hep-th/0607085].
- [45] I. Biswas, D. Gaiotto, S. Lahiri and S. Minwalla, “Supersymmetric states of $N=4$ Yang-Mills from giant gravitons,” JHEP **0712** (2007) 006 [arXiv:hep-th/0606087].
- [46] G. Mandal and N. V. Suryanarayana, “Counting 1/8-BPS dual-giants,” JHEP **0703** (2007) 031 [arXiv:hep-th/0606088].
- [47] S. Bhattacharyya and S. Minwalla, “Supersymmetric states in $M5/M2$ CFTs,” JHEP **0712** (2007) 004 [arXiv:hep-th/0702069].